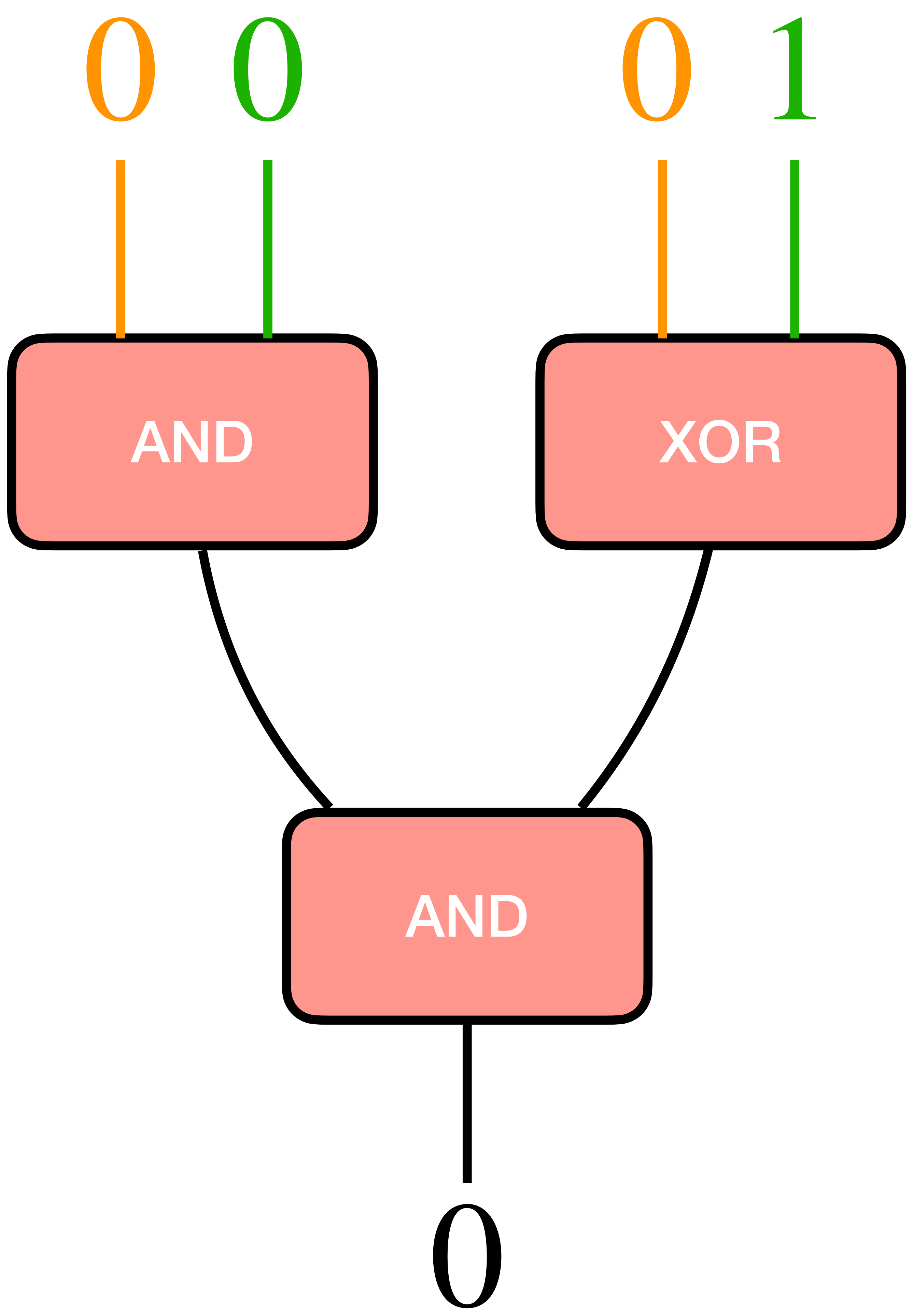
Garbled circuits

Recap

- Federated learning
  - Private summation of private model updates
  - Arithmetic secret sharing, Shamir secret sharing
- Password breach alert
  - Specialized private set intersection (PSI)
  - Oblivious PRF, DDH assumption

Garbled circuits

- Generic computation using a circuit-based computation model
  - Each party inputs a set of bits
  - Circuit made up of XOR and AND gates
  - Each gate has two input wire, and one output wire



Garbled circuits definitions

- Garble$(1^n, F) \rightarrow (GC_F, e, d)$: n is the security parameter, F is an input function to be garbled (represented as a boolean circuit), GC_F represents the garbled circuit for function F, e is the encoding information, and d is the decoding information

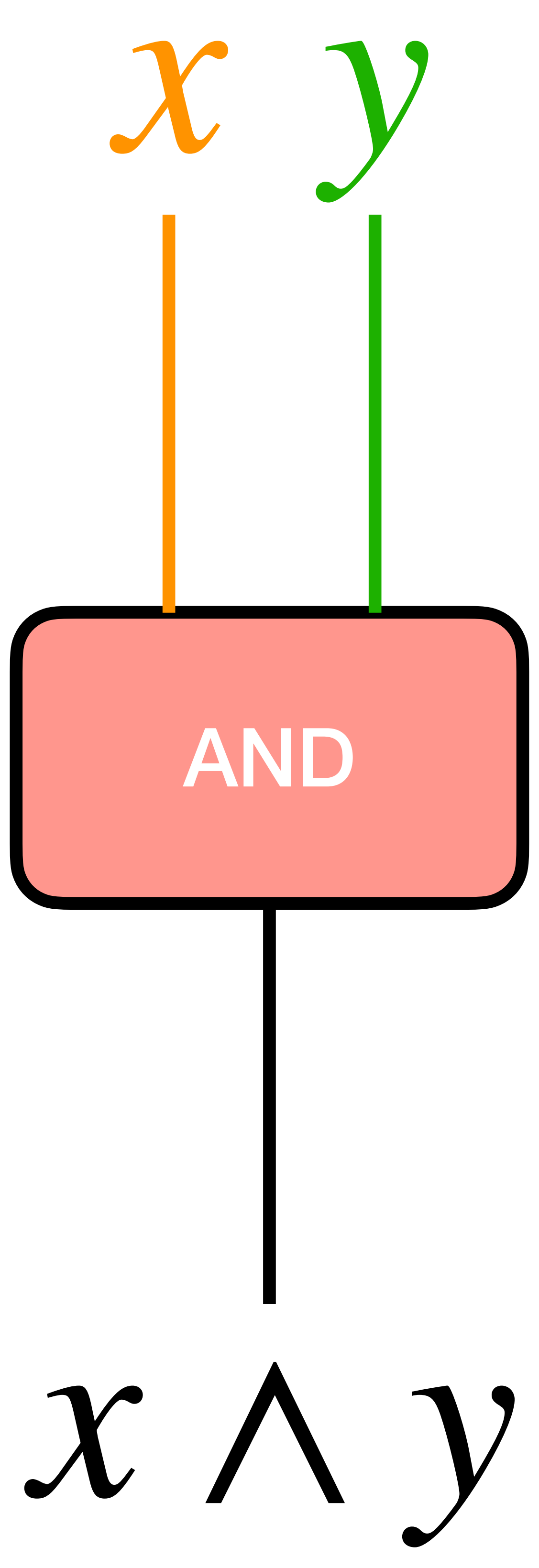
- Encode$(e, x) \rightarrow E_x$: k is a key, and x is corresponding input, E_x is the corresponding garbled input

- Eval$(GC_F, E_x) \rightarrow E_y$: on input a garbled circuit GC_F and an encrypted input E_x, produce a garbled output E_y

- Decode$(d, E_y) \rightarrow F(x)$: using decoding information d and garbled output E_y, output $y = F(x)$

Garbled circuits

- Alice & Bob want to figure out whether they should collaborate on a project, but doesn't want to reveal their own input
- Alice: x, Bob: y; want to compute $x \wedge y$ (circuit with a single gate)
- Two parties: Garbler (Alice) & Evaluator (Bob)
  - Garbler generates the circuit
  - Evaluator evaluates the circuit
- **Basic idea: encode the truth table of a gate using encryption**



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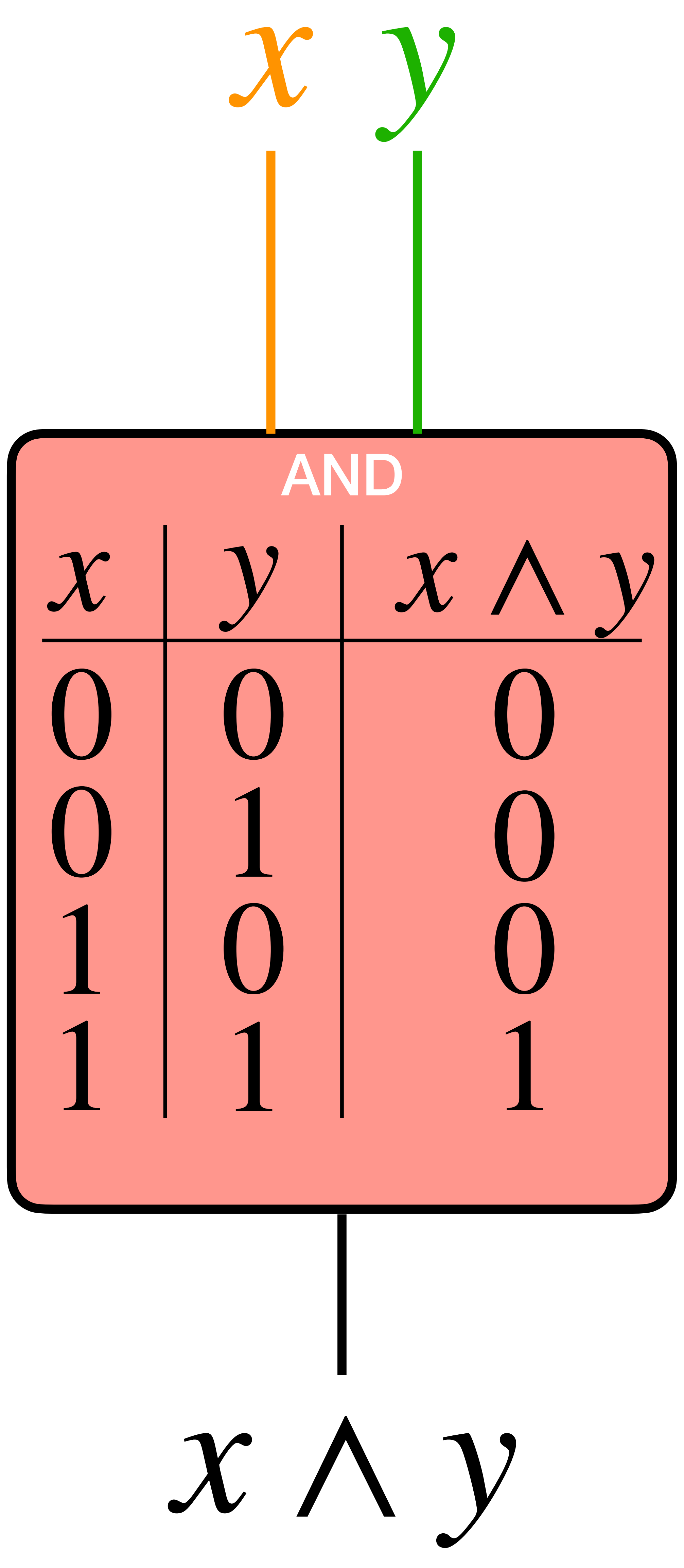
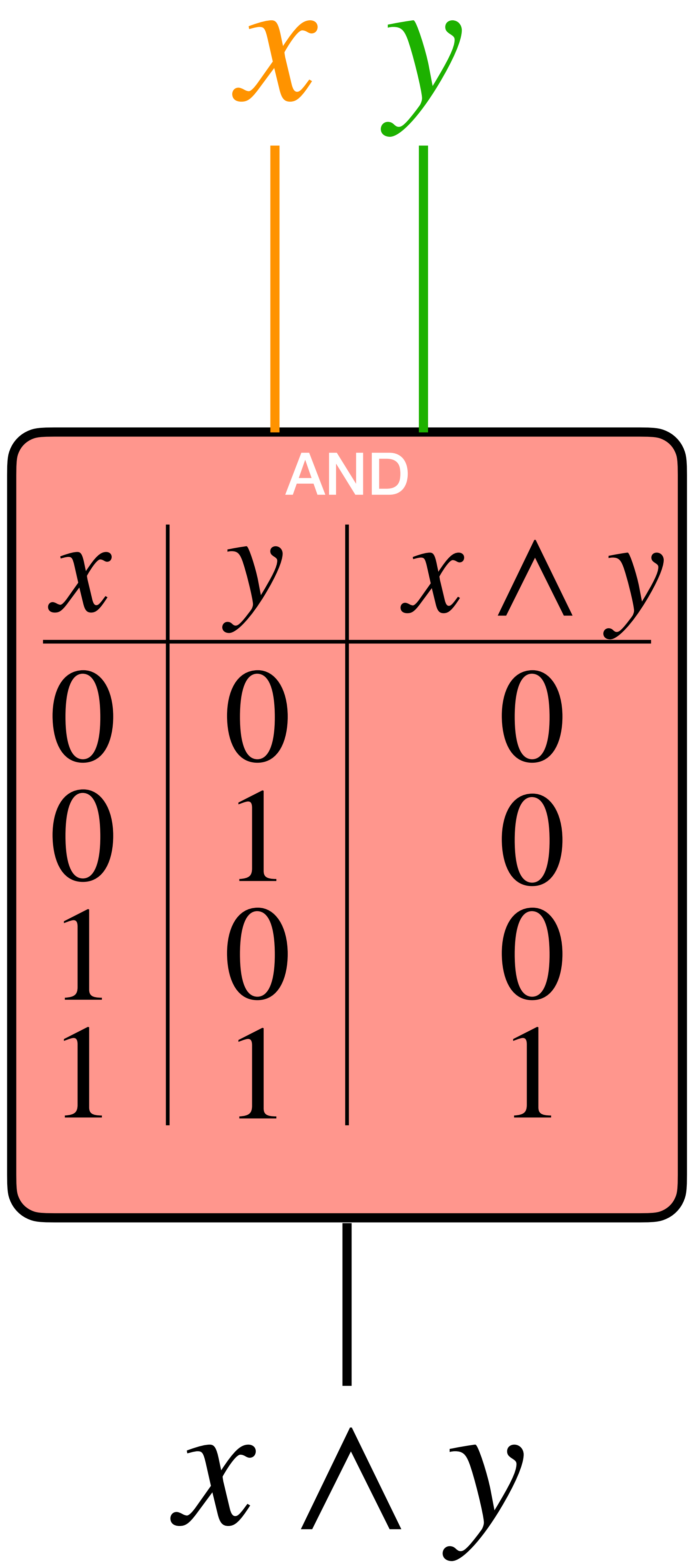


Garbled circuits: Garbler

- Let H be a key derivation function
- Pick four random labels: $W_x^0, W_x^1, W_y^0, W_y^1$, which correspond to the four possible values for x and y



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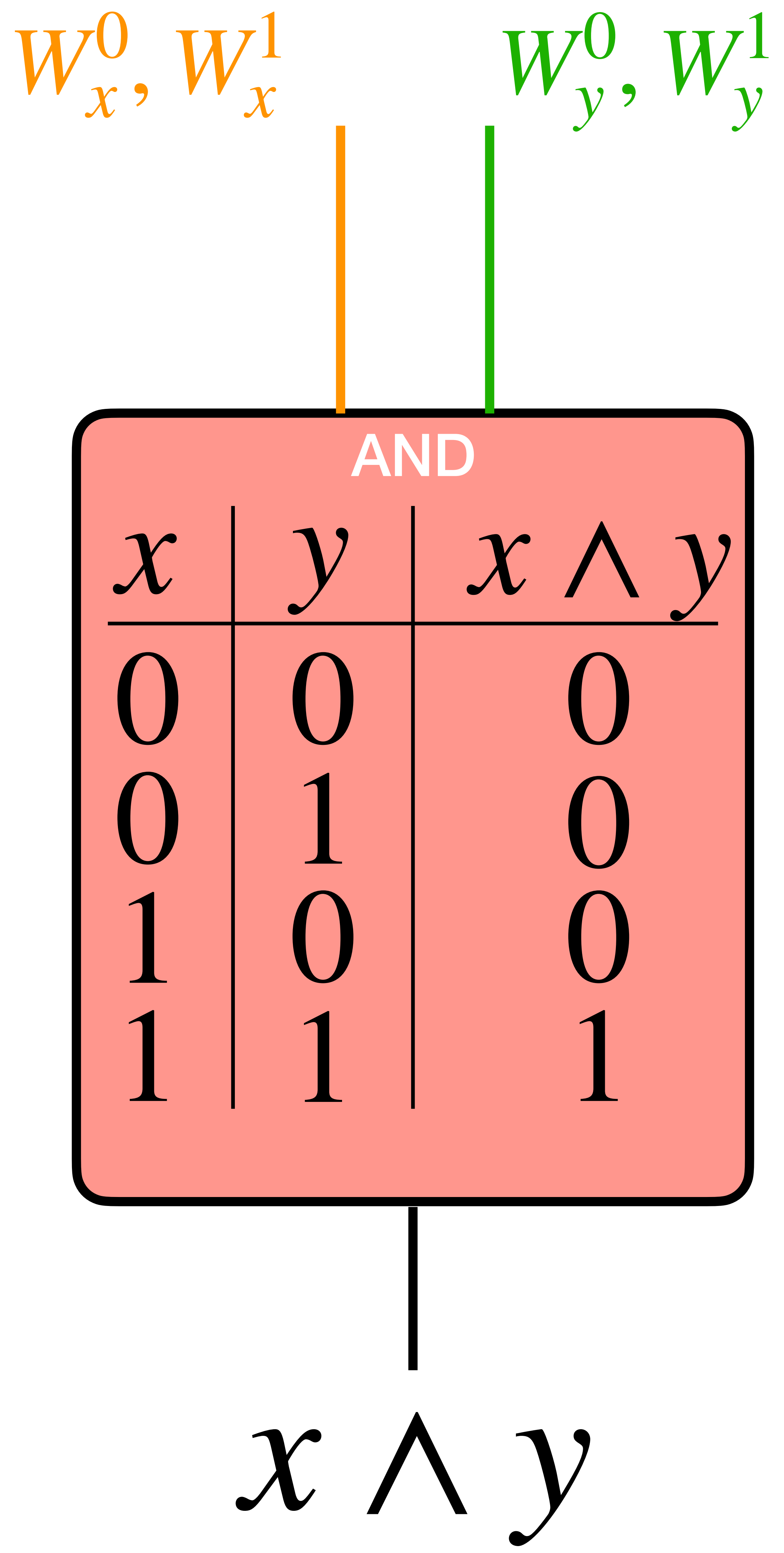

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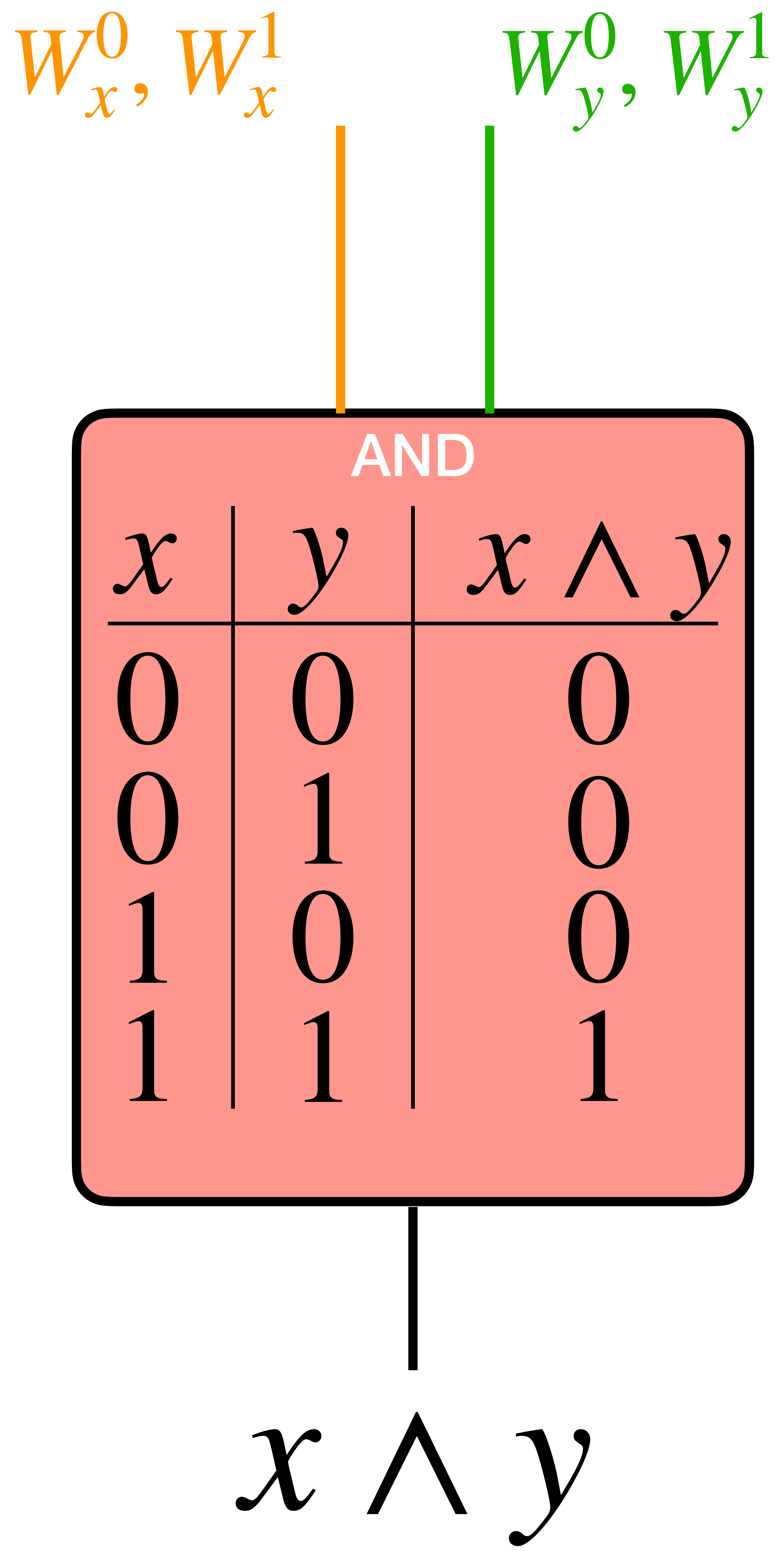
- For each row
  - Use H to derive a key using the corresponding labels
  - Encrypt the content



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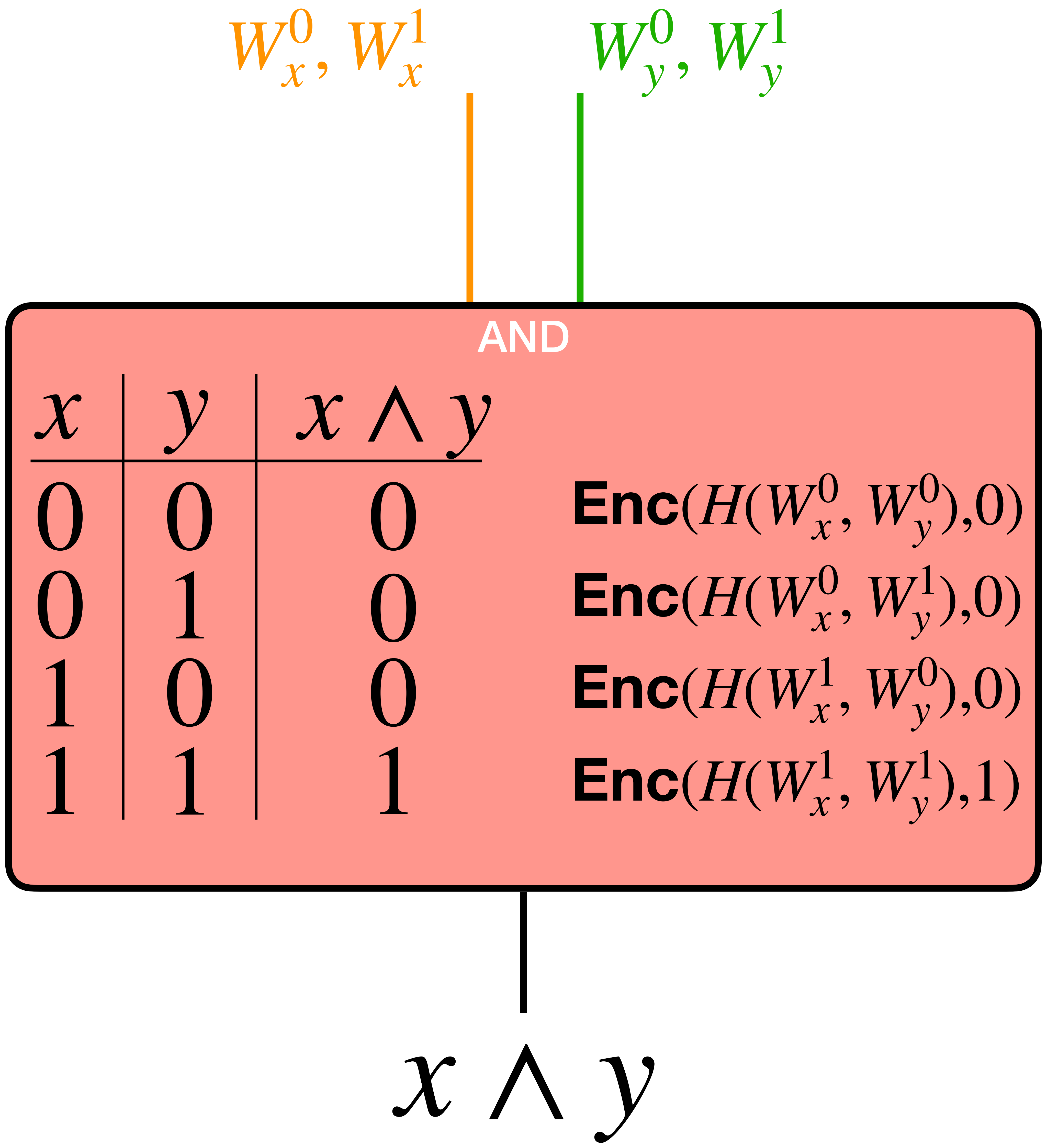

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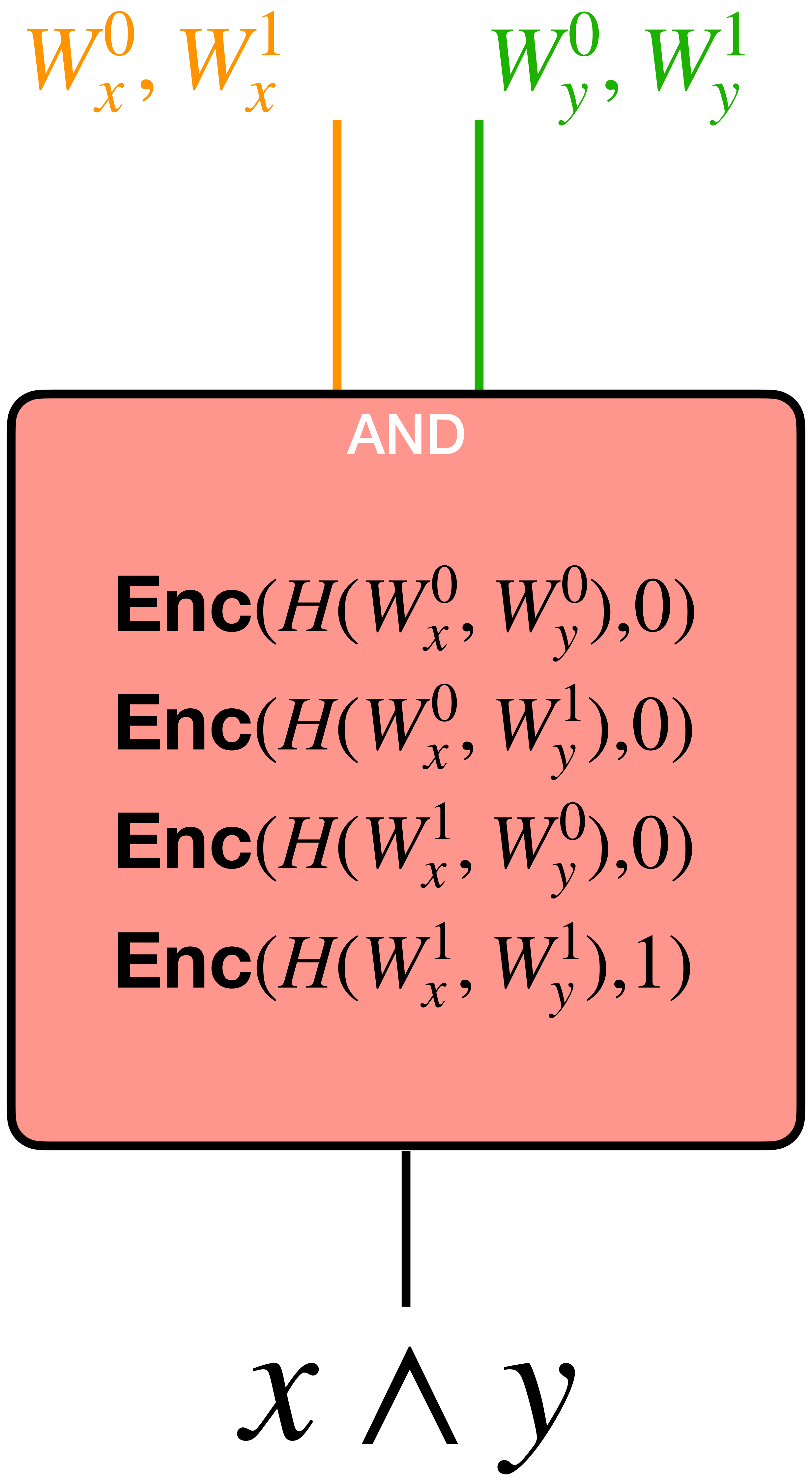
- Randomly permute the rows



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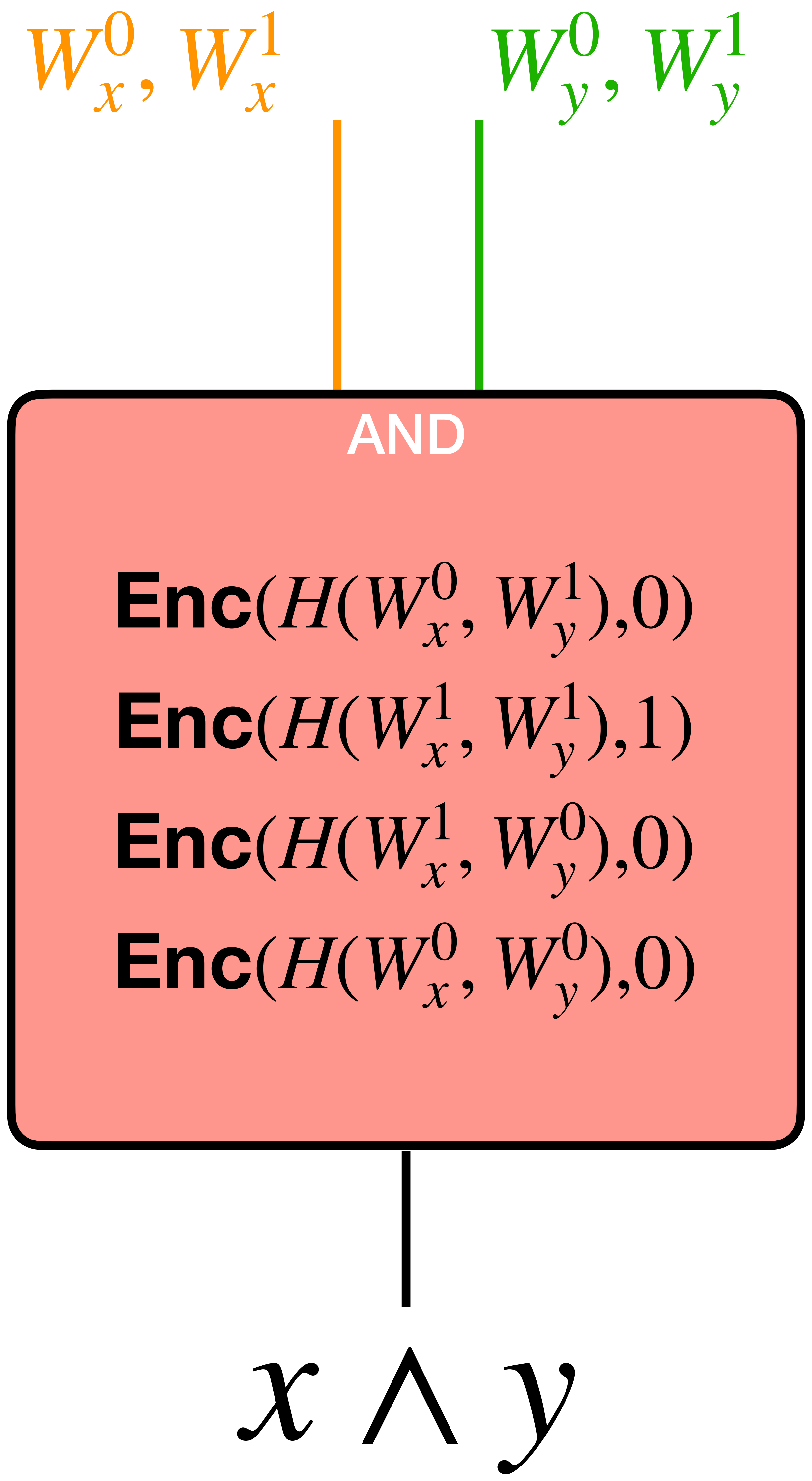
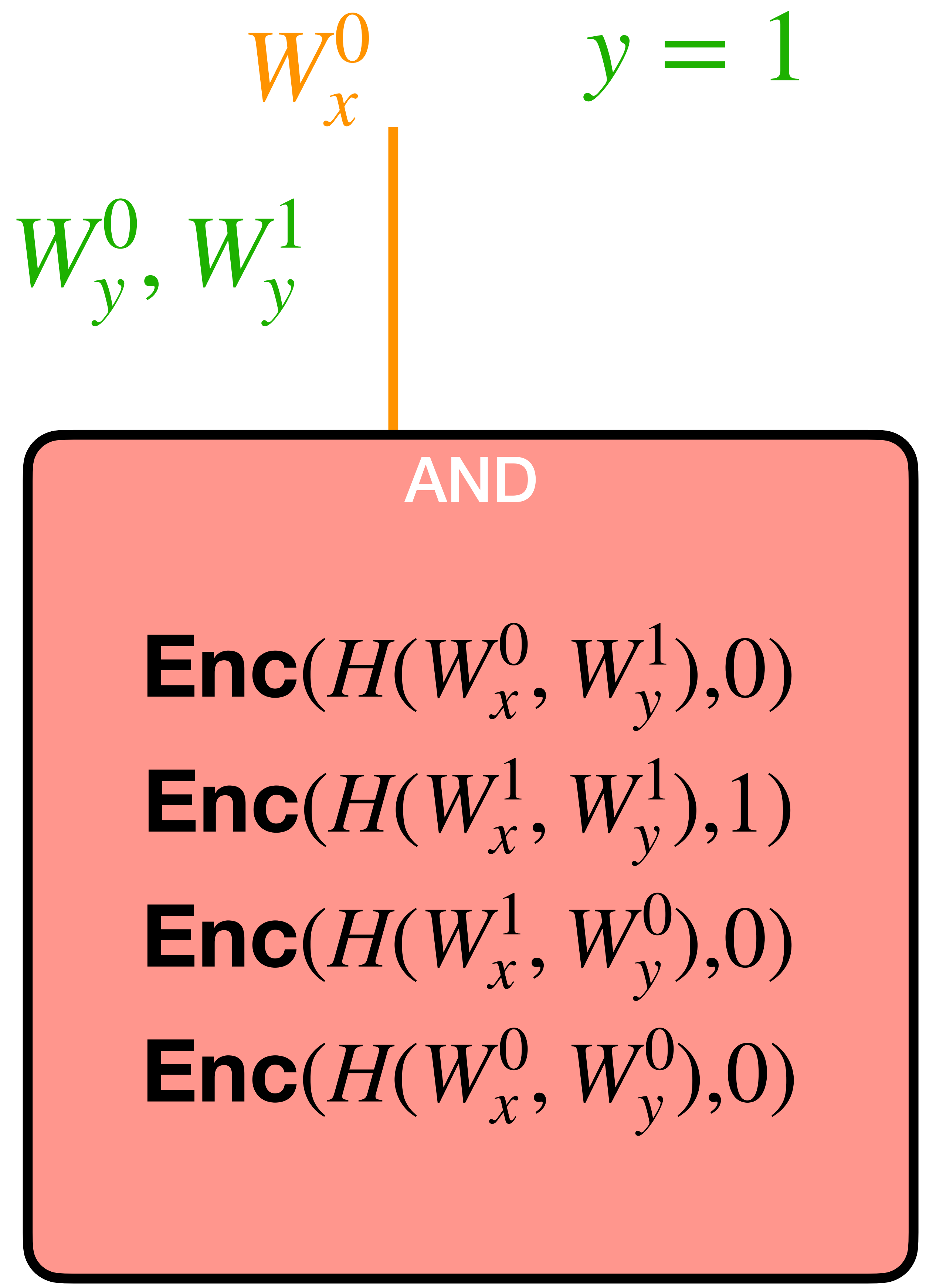


Garbled circuits: Evaluator

- In order to evaluate the gate, needs to know the correct label for each input wire
- Alice the Garbler can send her input wire label over directly — nothing is revealed since it's random
- What about Bob's value?
  - Alice should give the right label, without learning Bob's input
  - Bob should only learn one label, not two
  - Use **Oblivious Transfer**!



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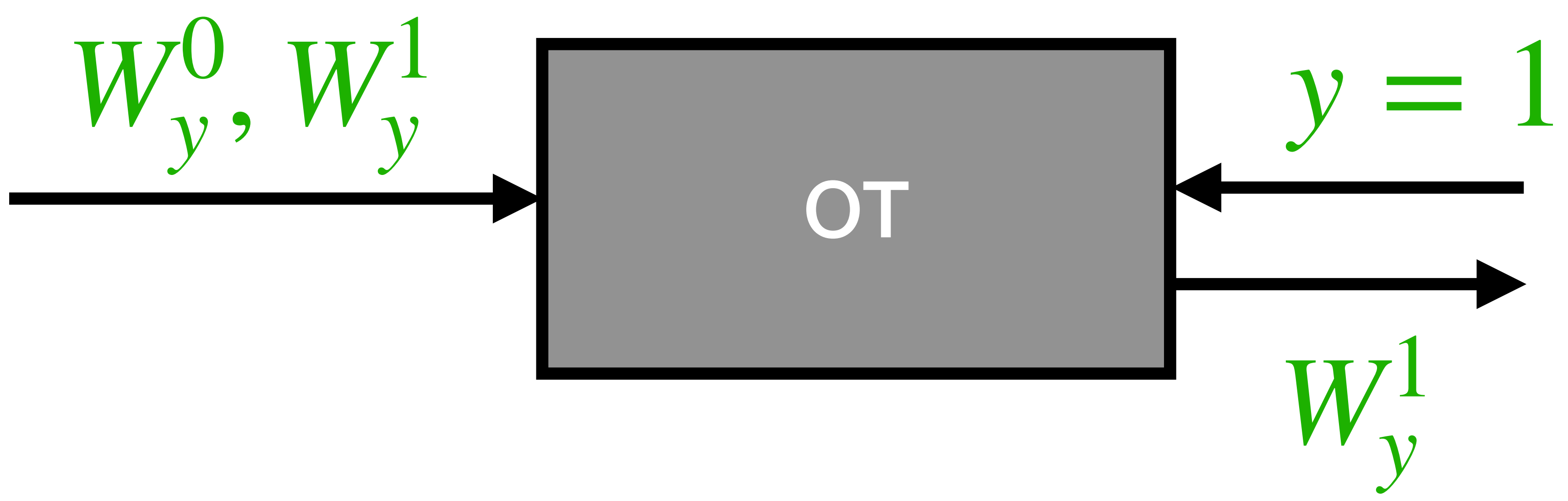

Garbler doesn't learn y
Evaluator doesn't learn W_y^0

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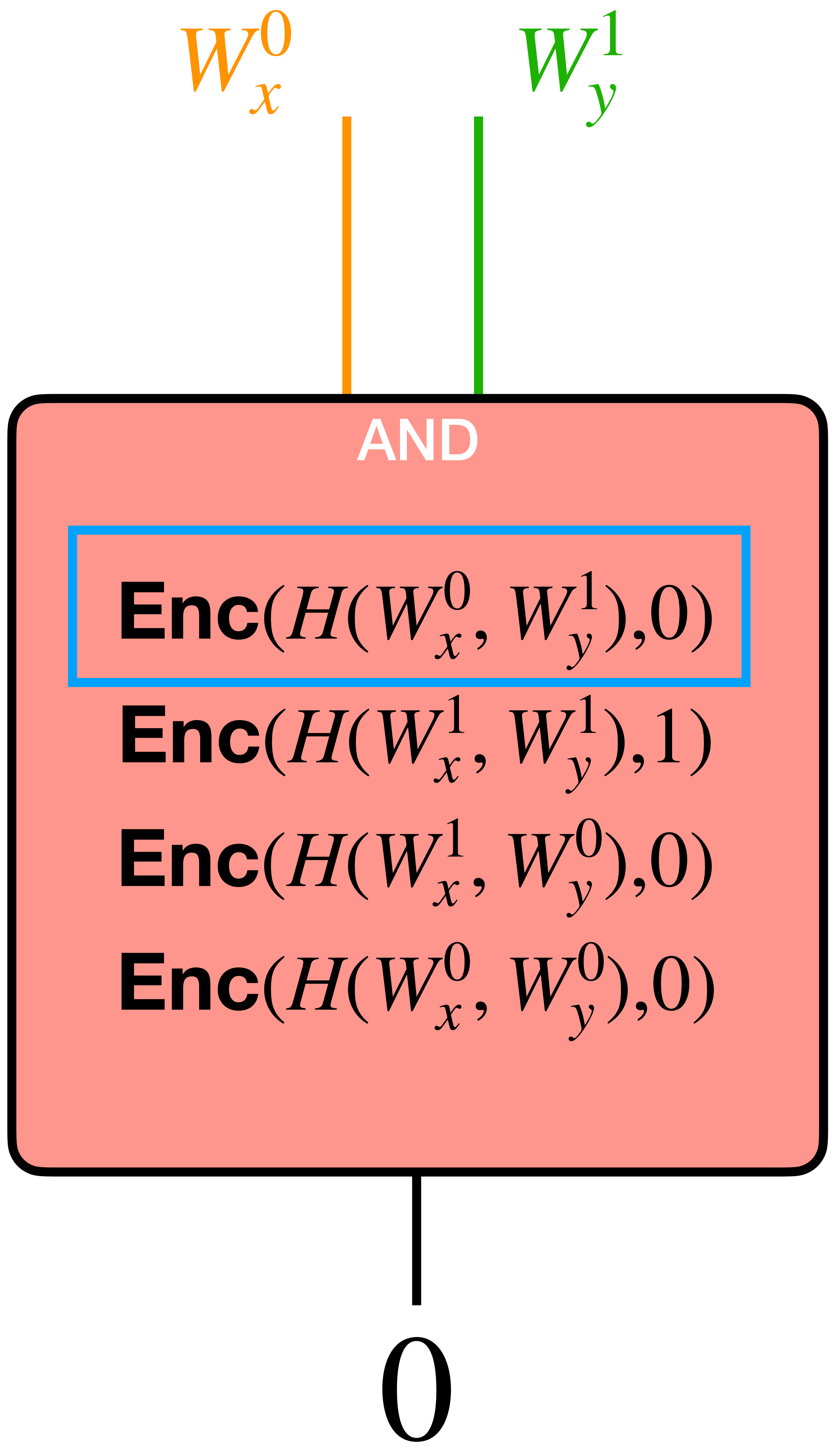
  - Use **Oblivious Transfer**!
- Use H to generate key, decrypt all four entries using key; if succeeds, output the result



Garbled circuits

- Extending to a circuit requires encrypting labels
- Security
  - Semihonest construction (otherwise garbler could choose an incorrect circuit, which requires other techniques)
  - Garbler is corrupted: security of OT
  - Evaluator is corrupted:
    - Labels are random
    - Permutation ensures no information is leaked from the organization of the circuit
    - Only one label is learned per wire

Today’s reading: SecureML

Next class

- 2-party, convolutional neural network inference
- Setup:
    - Server provides model
    - Client provides input
    - Client wants to run inference on the model without revealing the input; server does not want to reveal the model
- Techniques: HE (linear) & GC (non-linear)