Private information retrieval

Slides adapted from here, here, here

Last class: FSS & DPF

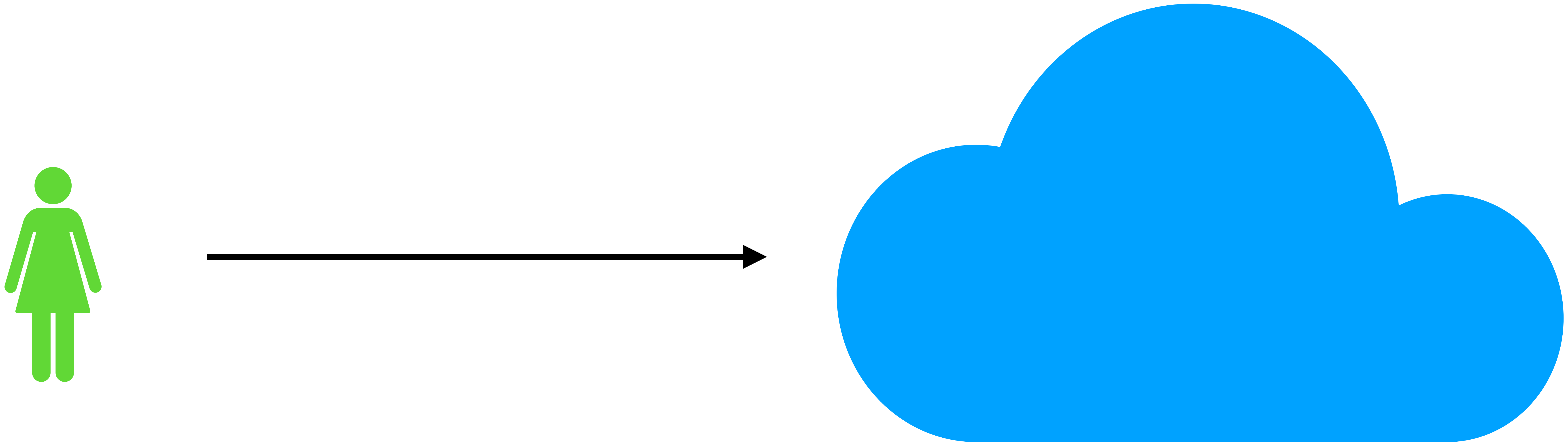
- Function secret sharing: allows a dealer to split a function f into function shares f_i such that for any input x, $f(x) = \sum_i^n f_i(x)$, where f_i are *succinct* and *secret*

- Distributed point functions: a special function that can be efficiently shared
  - Define a point function $f_{\alpha,\beta} : \{0,1\}^n \to \mathbb{G}$ for $\alpha = \in \{0,1\}^n$ and $\beta \in \mathbb{G}$ where $f(\alpha) = \beta$, and $f(x) = 0$ for $x \neq \alpha$

- Setting: multiple servers with some collusion threshold, each holding a copy of the full dataset

Private information retrieval (PIR)

“Can a user query a database without the database learning the query?”



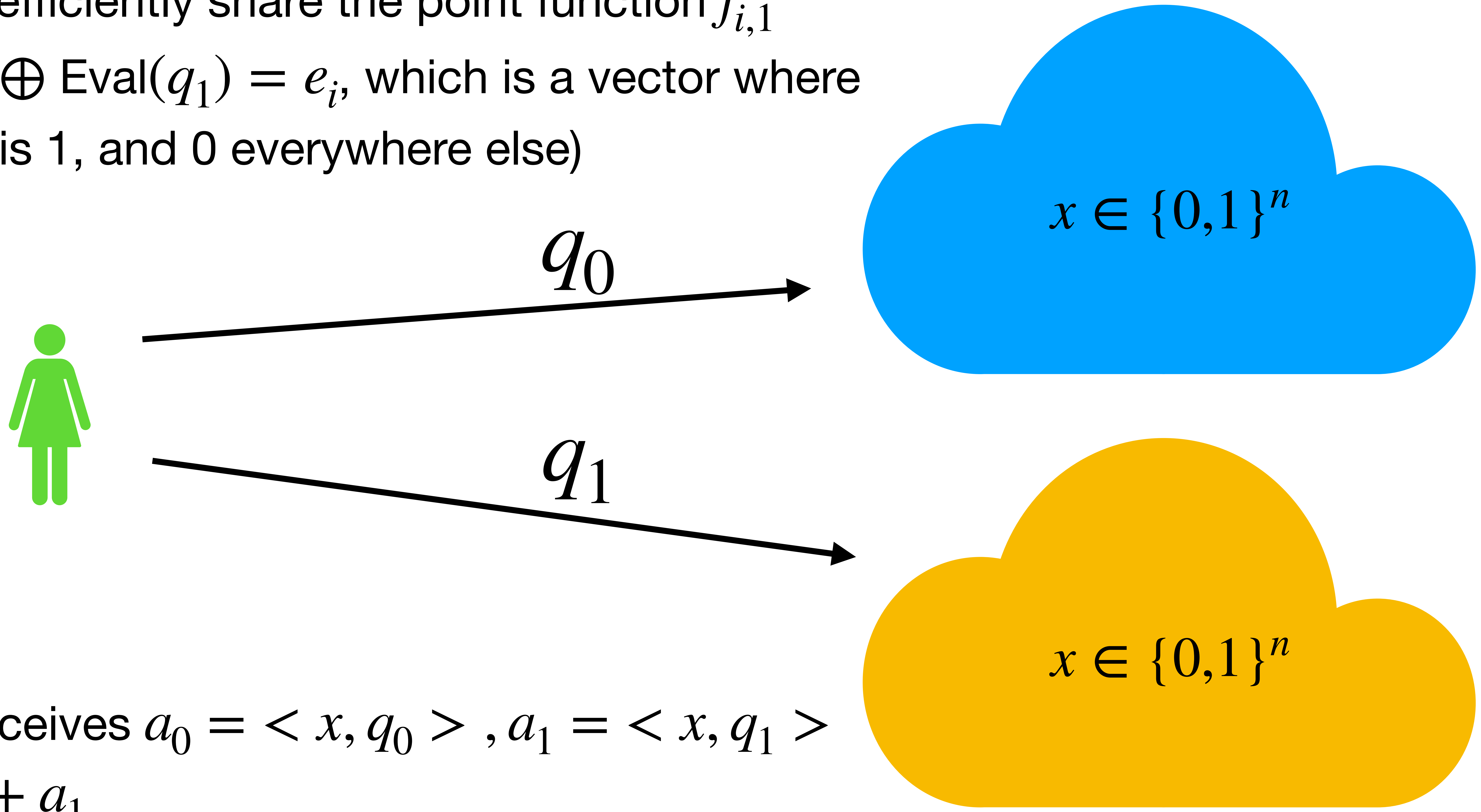
Many potential applications: DNS lookup, keyword searching, etc.

PIR definitions

- Database x of n bits
  - Query$(1^n, i) \to q$, Answer$(x, q) \to a$, Decode$(a) \to x_i$
- **Correctness:** client gets the bit that it wants
  - $\forall n \in \mathbb{N}, \forall i \in [n], \forall x \in \{0,1\}^n$,
    $Pr[\text{Decode}(a) = x_i : q \leftarrow \text{Query}(1^n, i), a \leftarrow \text{Answer}] = 1$
- **Privacy:** server should not learn anything about client's bit
  - $\forall n \in \mathbb{N}, \forall i, i' \in [n], \{q \leftarrow \text{Query}(1^n, i)\} \approx_c \{q \leftarrow \text{Query}(1^n, i')\}$

PIR via DPFs

DPF can efficiently share the point function $f_{i,1}$ ($\text{Eval}(q_0) \oplus \text{Eval}(q_1) = e_i$, which is a vector where ith index is 1, and 0 everywhere else)



Client receives $a_0 = < x, q_0 >$, $a_1 = < x, q_1 >$

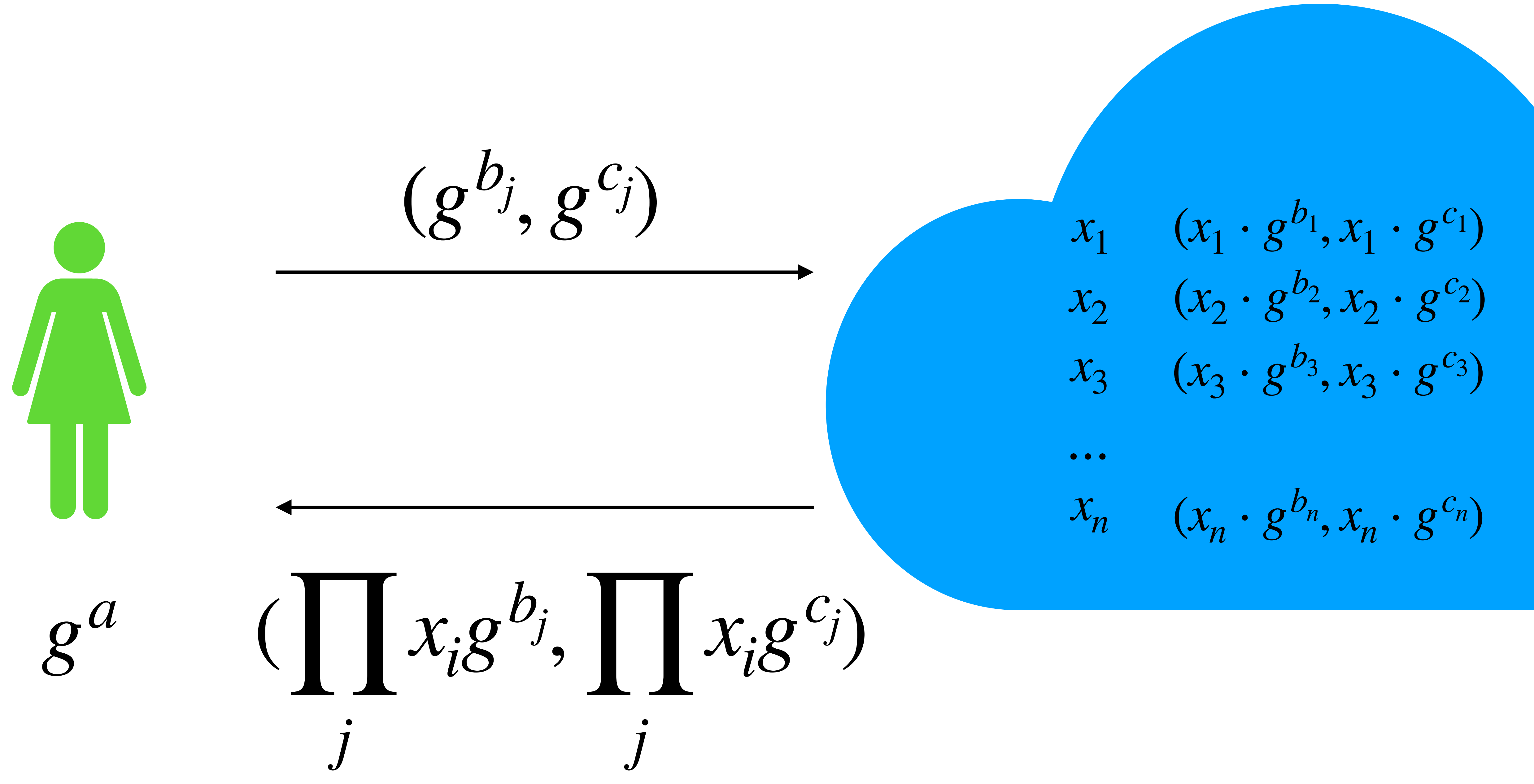
$x_i = a_0 + a_1$

Single server PIR

- Recap: the DDH problem
- Let $\mathbb{G}$ be a cyclic group of prime order q generated by $g \in \mathbb{G}$
  - Challenger computes $\alpha, \beta, \gamma \leftarrow \mathbb{Z}_q, u \leftarrow g^\alpha, v \leftarrow g^\beta, w_0 \leftarrow g^{\alpha\beta}, w_1 \leftarrow g^\gamma$
  - $(u, v, w_0) = (g^\alpha, g^\beta, g^{\alpha\beta})$ is a Diffie-Hellman tuple
  - Challenger gives (u, v, w_b) to the adversary where $b \leftarrow \{0,1\}$
  - Hard for adversary to guess $\hat{b} = b$
- An extra property: given a DH tuple (u, v_1, w_1), a tuple (u, v_2, w_2), then $(u, v_1 \cdot v_2, w_1 \cdot w_2)$ is a DH tuple if and only if (u, v_2, w_2) is a DH tuple

Single server PIR

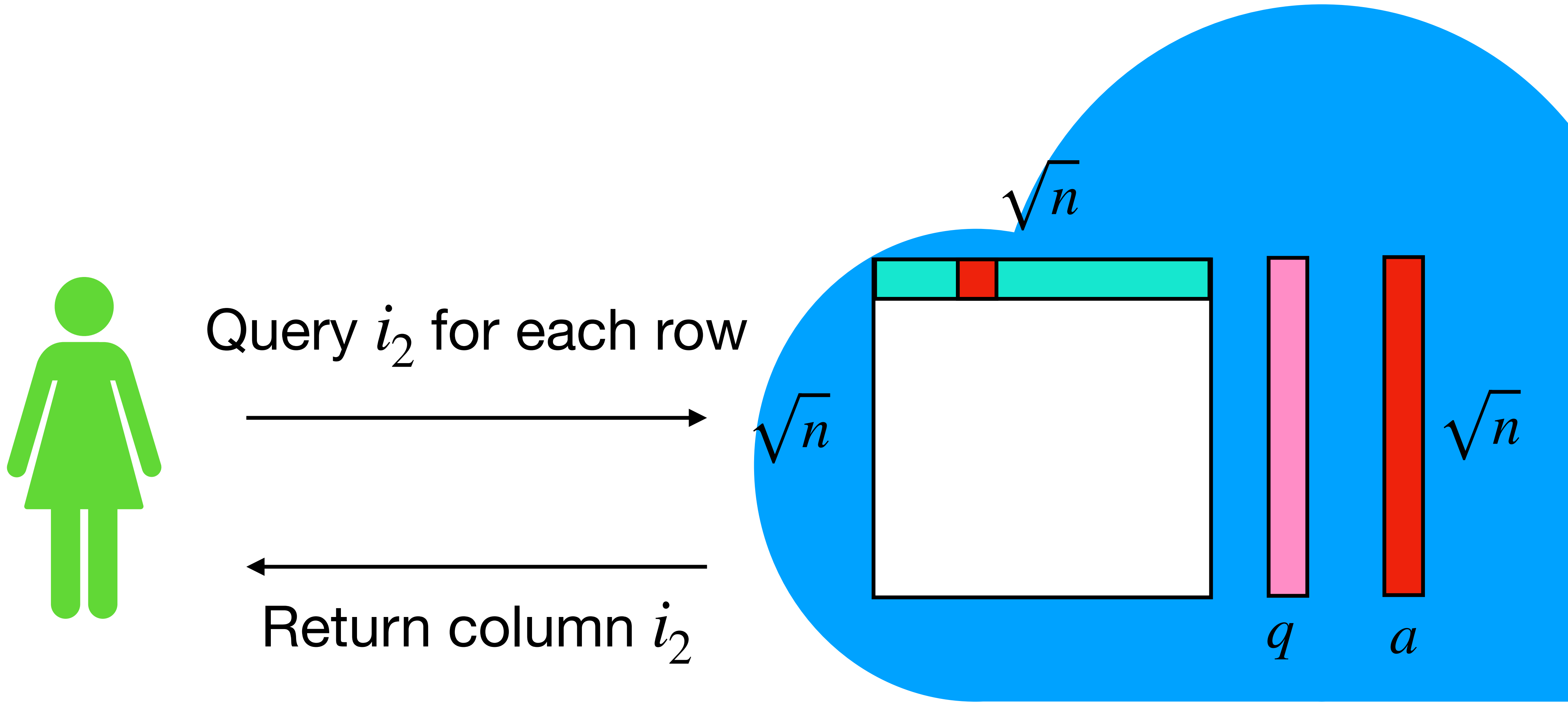
- Server holds database $x \in \{0,1\}^n$
- Client inputs an index $i \in \{1,\cdots,n\}$
- To query an index i
  - Client prepares n triples where the ith tuple is a non-DH tuple
    - Constructs g^a, g^{b_j}, g^{c_j}, for $j = 1,\cdots,n$
    - $c_j = ab_j$ for $j \neq i$, otherwise choose random c_j
  - Server computes and sends $\prod x_j g^{b_j}$ and $\prod x_j g^{c_j}$ (dot product)
  - If $(g^a, \prod x_j g^{b_j}, \prod x_j g^{c_j})$ is a DH tuple, then $x_i = 0$, otherwise $x_i = 1$



$O(n)$ query, $O(1)$ answer

Single server PIR with better communication

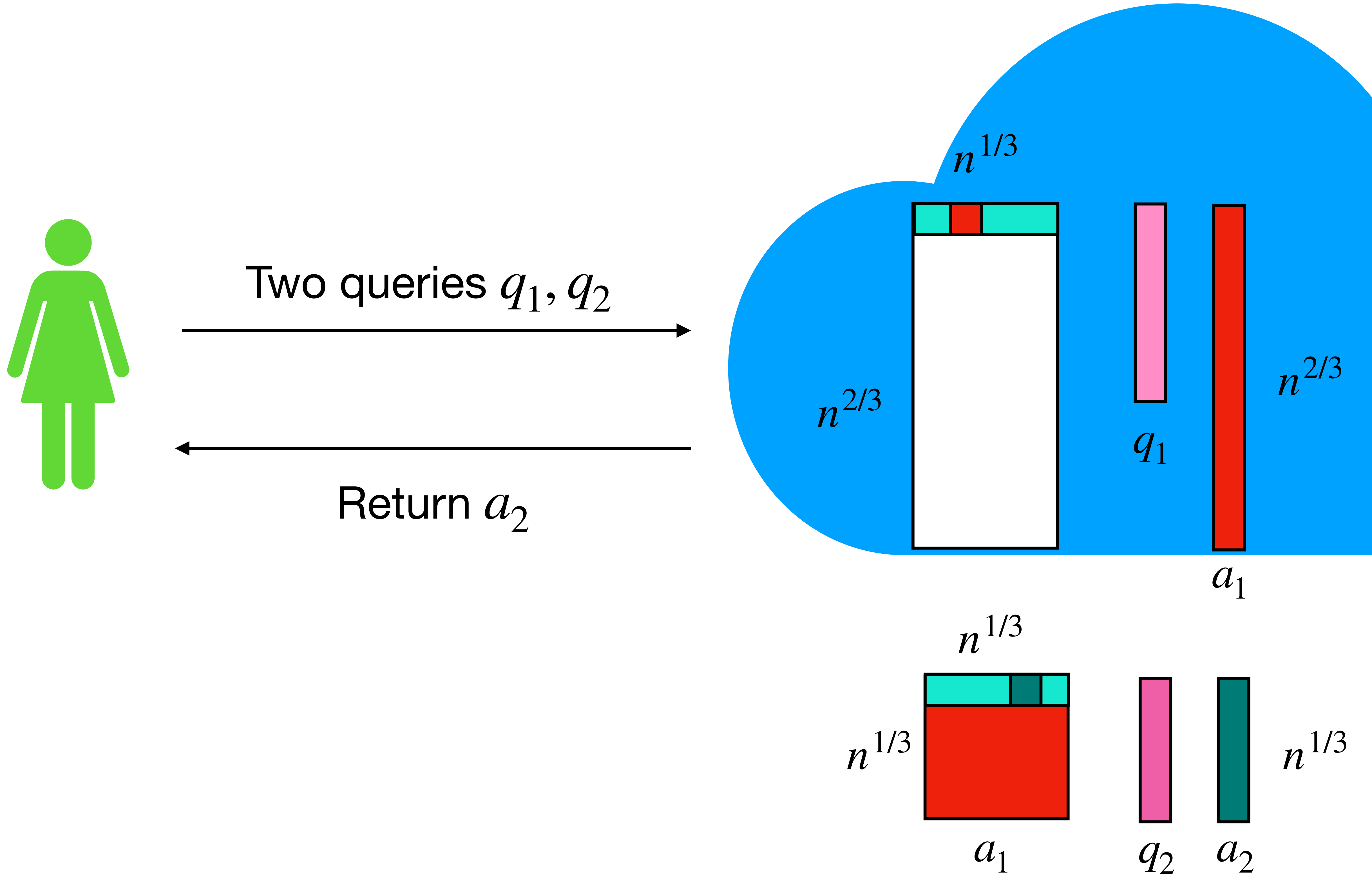
- Tradeoff between query length and answer length
- Restructure the database and view is as a matrix of size $\sqrt{n} \times \sqrt{n}$
- Bit i is represented (i_1, i_2), an element in the matrix
- Client constructs a PIR query with index i_2
- Server applies PIR on each row, returns one column (matrix multiplication)
- Client chooses the i_1-th item



$O(\sqrt{n})$ query, $O(\sqrt{n})$ answer

Even better communication

- Insight: though the answer is of length $\sqrt{n}$, the client only needs one element
- Idea: can view the answer to the query as *another database* and run a second PIR on this DB!
- Recursion results in a complexity that is asymptotically smaller than n^{ϵ} for every constant $\epsilon > 0$
- Tradeoff is more compute



ORAM	PIR
Memory contents changes with every query	Public, static DB
One client → one server	Multiple clients → one server
Reads and writes	Traditionally only for reads
Server process in $polylog(n)$	Linear server work per query

PIR is still expensive

- Communication cost
  - Two-server PIR: $O(\log n)$
  - Single-server PIR: $polylog(n)$ from public key crypto assumptions
- Computation cost
  - Batching: batch multiple queries together in a single scan
  - Preprocessing: by offloading some work in a separate preprocessing phase, and by storing extra information, the “online” cost of a retrieval is less than a linear scan

Today’s reading: Pung

Next time: Vuvuzela

- A very different approach to anonymous messaging
- No longer using a database abstraction
- Do not need to use heavy crypto -> much more scalable
- Network traffic & dead drop access patterns leak information
    - Same chain of servers used to shuffle traffic & add cover traffic (all but one can be compromised)
    - Differential privacy offers a scalable way hiding metadata (albeit weaker)